Technical Report No. 2
NGR-23-004-085
NASA Langley Research Center
Hampton, Virginia 23365

APPLICATION OF WAVE MECHANICS THEORY TO FLUID DYNAMICS PROBLEMS--FLAT PLATE FLOW

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Division of Engineering Research
MICHIGAN STATE UNIVERSITY
East Lansing, Michigan 48824
October 31, 1974

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APPLICATION OF WAVE MECHANICS THEORY TO FLUID DYNAMICS PROBLEMS -- FLAT PLATE FLOW

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1. FLAT PLATE FLOW

1.1. Introductory Remarks

Technical Report No. 2 contains only the results of the research on the flow in the laminar boundary layer along an infinitely long flat plate. The flow may be disturbed or not, depending upon the situation. It is always mentioned very clearly whether the flow is disturbed and the disturbing function is always described.

1.2. Laminar Flow Along an Infinitely Long Flat Plate

The flow in question is very thoroughly described in Technical Report No. 1. Consequently, the description does not need to be repeated here. The writer passes directly to the explanation of the plates, oscillograms and plots. These are divided into certain number of groups, depending upon the origin of the picture, the plate or oscillogram and its meaning.

Group No. 1 refers to the physical, natural aspects of the flow, either a laminar free from disturbances or a flow which originally is a laminar one and next disturbances are superimposed upon it.

Plate No. 19. Four oscillograms of turbulence in windtunnel and in the wake of a cylinder. These pictures are taken from actual physical tests. They are cited here intentionally for the purpose of comparison.

1a. First from the top: Turbulence in windtunnel, Time 0.4 sec., approximately. Relative Amplification = 64.

1b. Second from the top: Center of Wake; 2 1/2" behind 13/16" cylinder; Relative Amplification = 1; Time = 0.3 sec., approximately.

1c. Third from the top: 13/32" Laterally from Center of Wake; 2 1/2" behind 13/16" cylinder; Relative Amplification = 1; Time = 0.3 sec., approximately.

1d. Fourth from the top: 1 1/2" Laterally from Center of Wake; 2 1/2" behind 13/16" cylinder; Relative Amplification = 8; Time = 0.3 sec., approximately.

Picture No. 15. Surface of a liquid medium covered by aluminum powder in turbulence.

Group No. 2 refers to laminar flow along an infinitely long flat plate with disturbances.

Plot No. 1. The horizontal velocity curve plus disturbance in the flow in the boundary layer along an infinitely long flat plate: $f'(\eta) + H'(\eta)$; $f'(\eta) = uU_\infty^{-1}$; $H'(\eta)$ = disturbance; $H'(\eta) = C_{add}\{\frac{1}{2}[\eta f''f'' + \ldots]\}$, is given in Technical Report No. 1, Equation (6.3.14); $C_{add} = 1$; $\eta^2 = y^2 U_\infty \nu^{-1} x^{-1}$; $y^2 = \eta^2 U_\infty^{-1} \nu x$; η^2 = temporarily fixed; the resultant parabola in (x, y) space is open to the right or left; points of intersections and the resultant path of a particle are clearly seen.

Plots No. 2, 3, 4: As above. $C_{add} = 2; =3; = 10$.

Plot No. 5. As above. $C_{add} = 30$.

The plots discussed below refer to the thermal boundary layer.

Plot No. 6. Temperature pattern + disturbance; $T(\eta) + T'(\eta)$; $T'(\eta)$ = disturbance; $T'(\eta)$ depends upon the Prandtl No., P, Eckert No., E, and the product $E \cdot B$ (P); in plot No. 6 $E \cdot B = 6$, $E = 7.186$, $B = 0.835$.

Plot No. 7. As above. $E \cdot B = 4$, $E = 4.790$, $B = 0.835$.

Plot No. 8. As above. $E \cdot B = 2$, $E = 2.395$, $B = 0.835$.

Plot No. 9. As above. $E \cdot B = 0$, $E = 0.0$, $B = 0.835$.

Plot No. 10. As above. $E \cdot B = -2.0$, $E = -2.395$, $B = 0.835$.

Plot No. 11. As above. $E \cdot B = -4.0$, $E = -4.790$, $B = 0.835$.

Plot No. 12. As above. $E \cdot B = 6.0$, $E = 1.5$, $B = 4.0$.

Plot No. 13. As above. $E \cdot B = -4.0$, $E = -1.0$, $B = 4.0$.

Plot No. 14. Enlargement of Plot No. 2, $C_{add} = 2.0$.

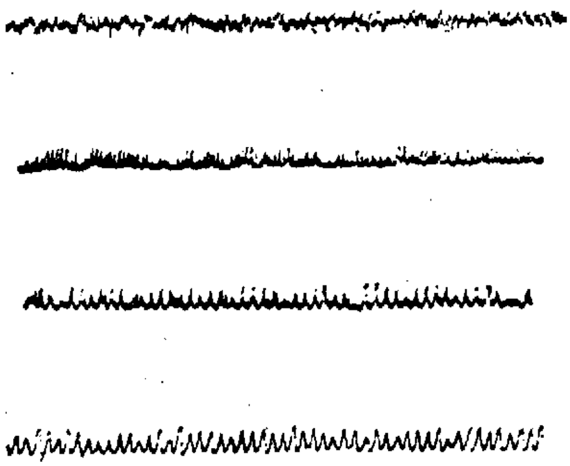
Group No. 3 deals with laminar flow with no disturbances. As demonstrated above, the so-called purely laminar flow from the macroscopic, deterministic point of view is a turbulent flow from the microscopic point of view. Some of the plots are repeated in Technical Report No. 1 as well as in the present report. This is done for the convenience of the reader.

2. DESCRIPTION OF PLOTS

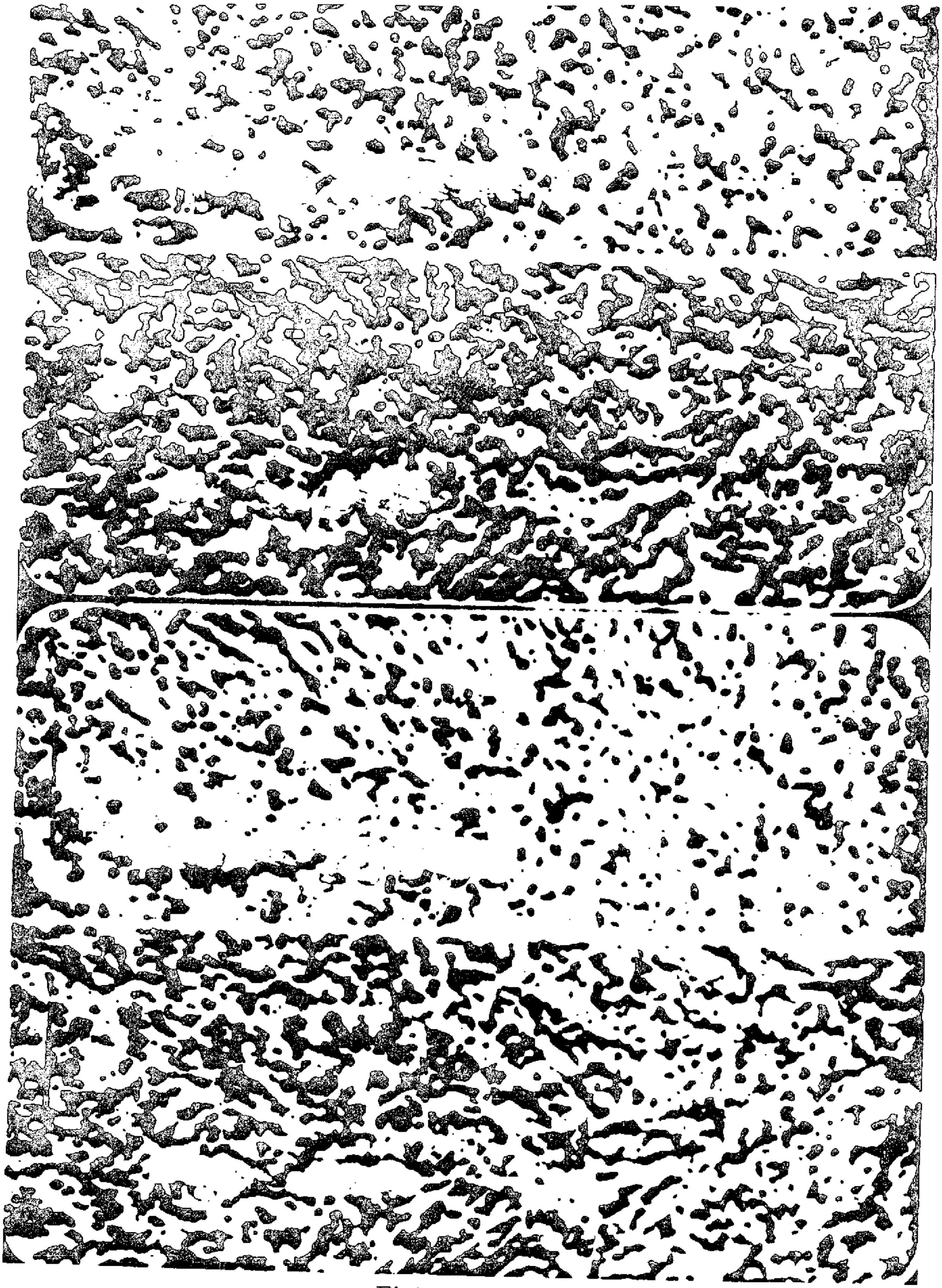
The plots refer to the flow in the boundary layer along an infinitely long flat plate. They allow one to see and to understand better the nature of turbulence, at least in some particular problems. The two plots given below are known from Technical Report No. 1. (1) Plot No. 21: The curve $f'(\eta) = uU_{\infty}^{-1}$ $(=UU_{\infty}^{-1})$ (Blasius 1908) can be distinctly seen on the left hand side of the plot. The composite coordinate η (ETA) is measured on the horizontal axis in the interval from $\eta = 0.0$ to $\eta = 8.8$ in the coordinate system. On the vertical axis, the values of the function $f'(\eta)$ as the function of η in the interval from $f'(\eta) = 0.00$ to $f'(\eta) = 1.00$ are measured. The resulting curve, $f'(\eta)$ = function of η, is the well-known (Blasius') curve. To complete the description, the function uU_{∞}^{-1} denotes the horizontal velocity component in the "laminar" boundary layer along an infinitely long flat plate in the macroscopic, deterministic fluid dynamics. Next, one chooses a number of points, η, on the curve $f'(\eta)$; at each of these points one assumes a fixed coordinate system (x, y). Through each point a parabola given in Equation (8.5.20) in Technical Report No. 1 is plotted. These parabolas are very flat, as explained above. The points of intersections of the parabolas (streamlines), connected together, form the zig-zag path of a cluster of particles. The zig-zag paths (unseen by the naked human eye in ordinary conditions) clearly seen on Plot No. 21, are done completely automatically by the plotter. (2) Plot No. 31: This is the same as Plot No. 21, but the parabolas are plotted according to the formula, $y^2 = 4ax$, Equation (8.5.22) (see Technical Report No. 1). The above two plots prove without doubt that the boundary layer, laminar from the deterministic, macroscopic point of view, is a turbulent one from the microscopic, wave mechanics point of view.

Group No. 4 pertains to composite flows. This group of plots represents a composite flow such as a flow between two parallel flat plates which are supposed to be infinitely long. Due to the linear character of the

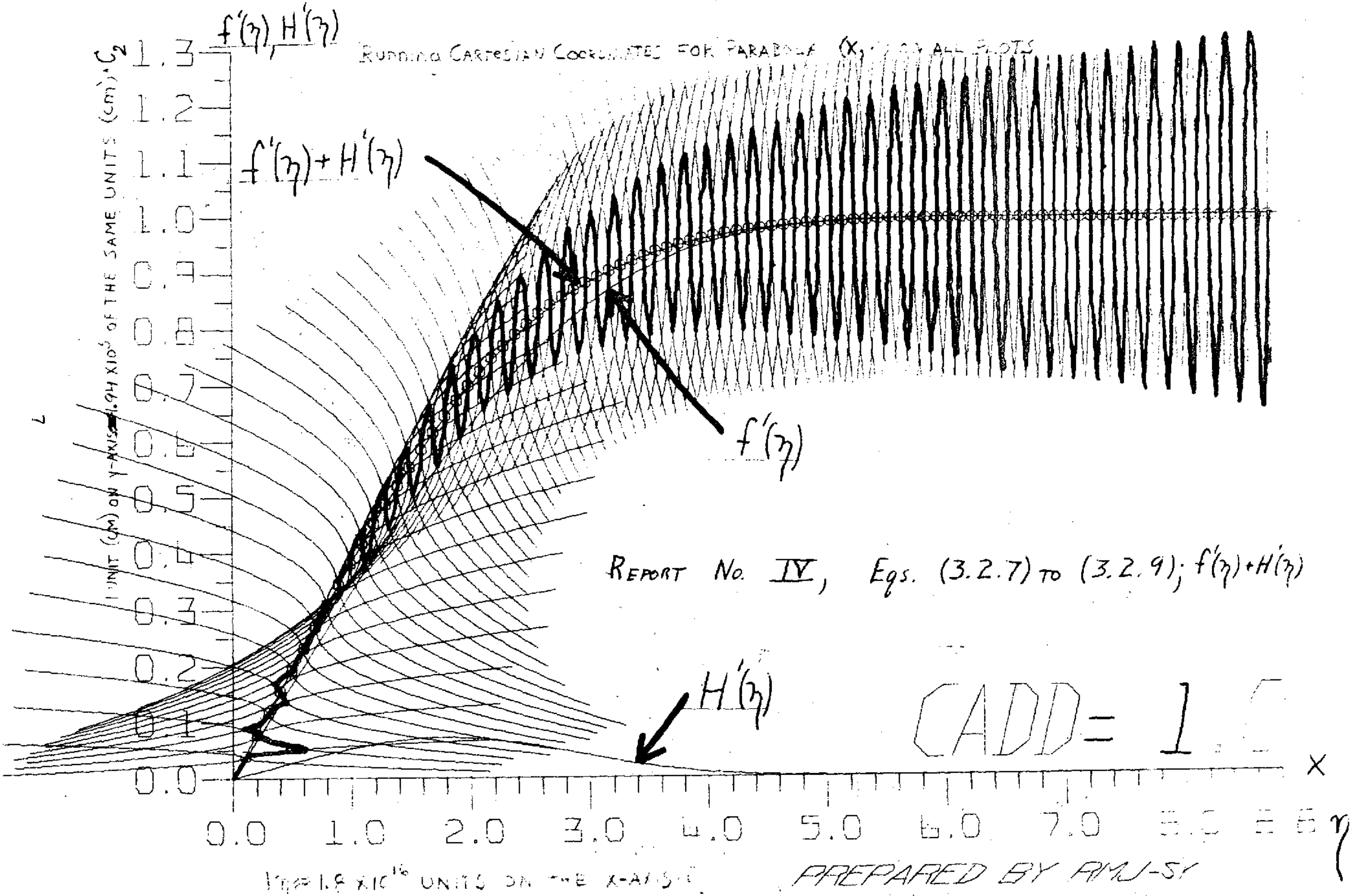
wave equation, any number of wave equations and their solutions can be summed. Imagine a laminar flow between two parallel, infinitely long flat plates and assume that these two flows do not interfer with one another. In this case, one can construct the known flow pattern in each flow including the streamlines and the zig-zag paths. This is shown on the two composite plots included here. Next, assume that the two flows interfer with one another. The set of streamlines in one laminar flow affects the flow and its structure due to the boundary layer along the second parallel, infinitely long flat plate. Briefly stated, each flow in the laminar boundary layer becomes a flow with disturbances superimposed upon it; i.e., $f'(\eta) + H'(\eta)$. Each laminar flow interfers with another laminar flow and the result of this interference is a disturbance in the laminar flow along both flat plates. Various phases of these disturbances are represented by the last five pictures.



Picture No. 19

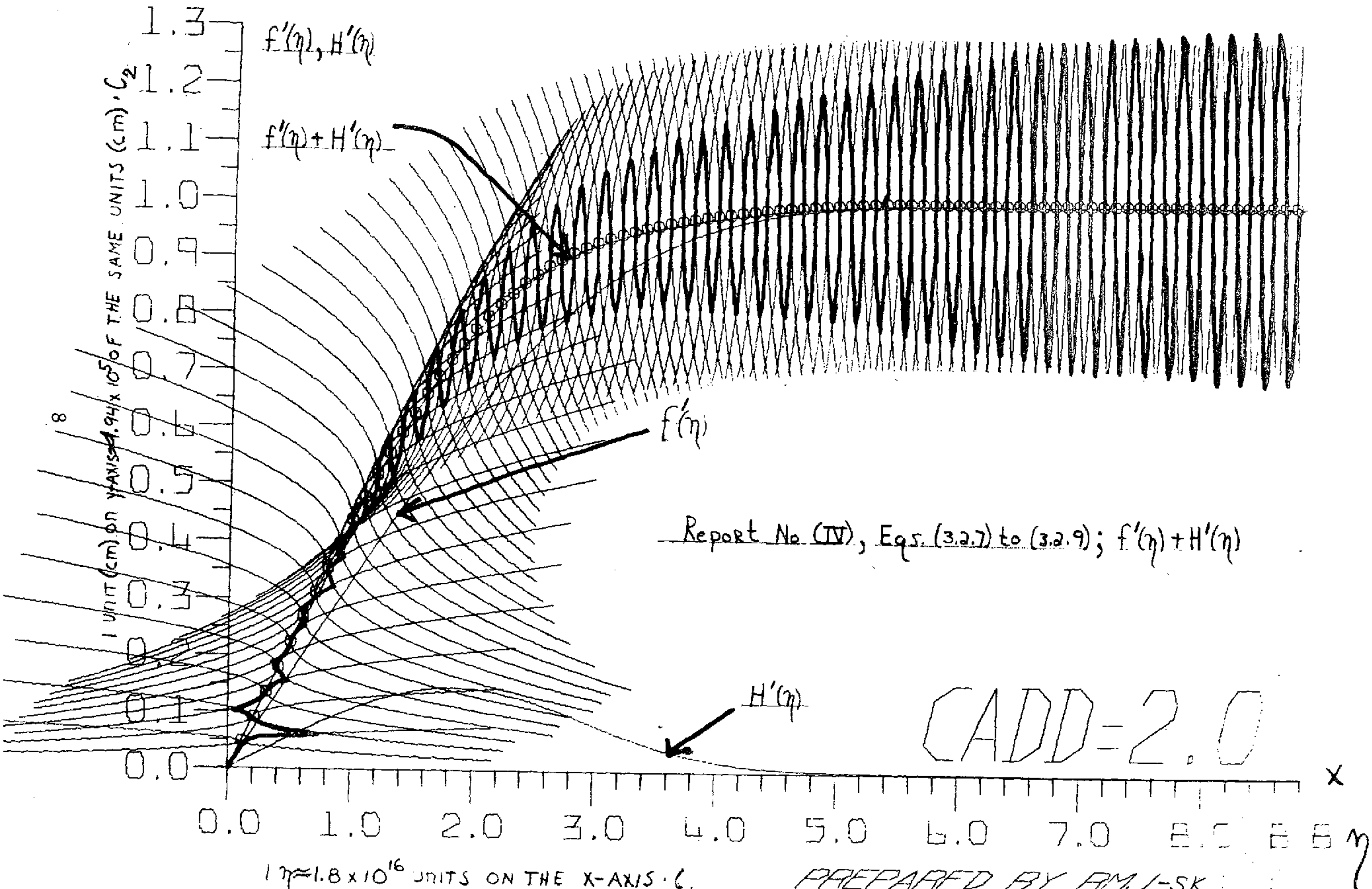


Picture No. 15.



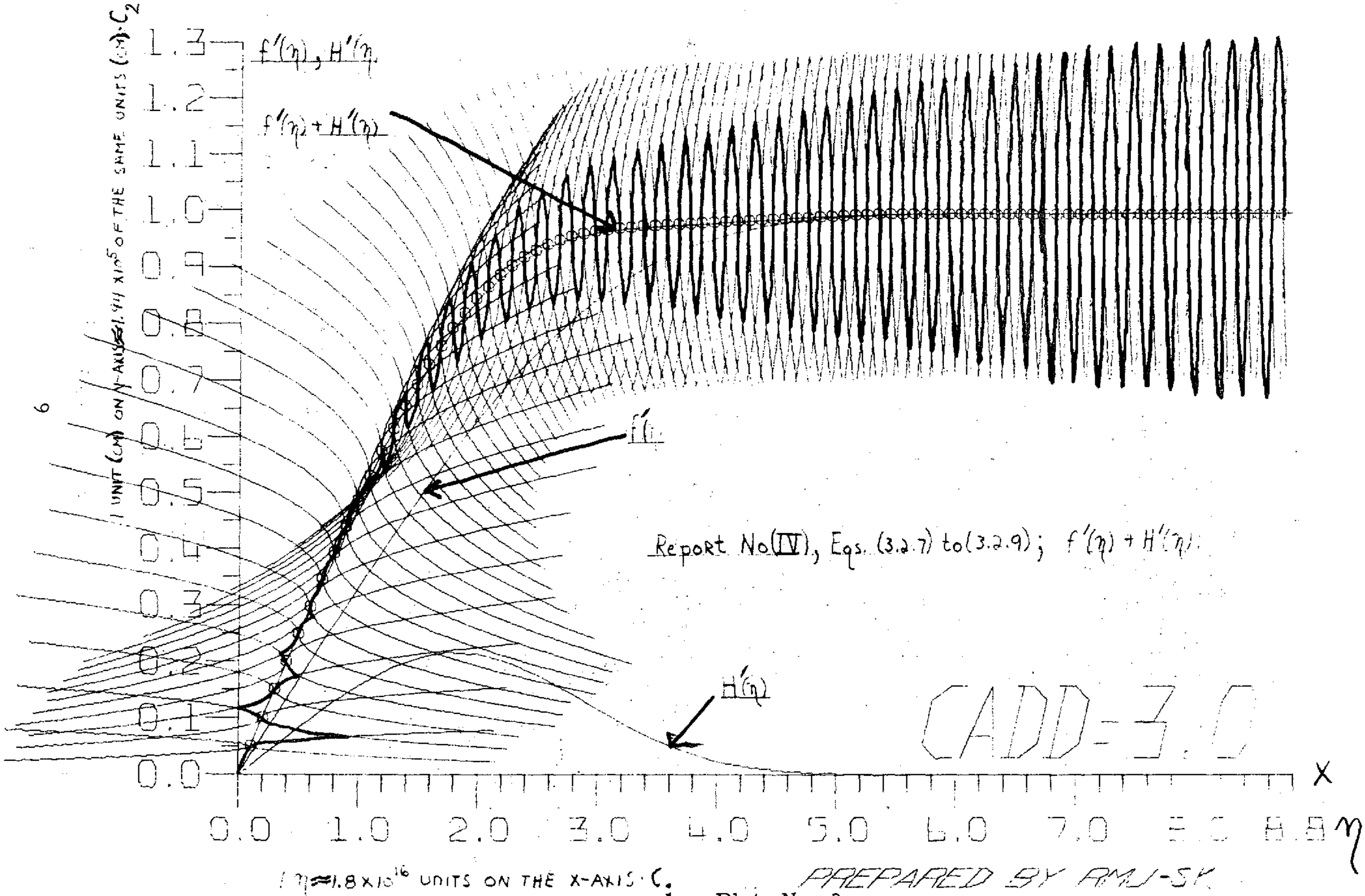
f'(η), H'(η)
f'(η)+H'(η)
f'(η)
H'(η)
REPORT No. IV, Eqs. (3.2.7) TO (3.2.9); f'(η)+H'(η)
CADD= 1.
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0
X
η
PREPARED BY RMJ-SY

Plate No. 1



f'(η), H'(η)
f'(η) + H'(η)
f'(η)
H'(η)
Report No (IV), Eqs. (3.2.7) to (3.2.9); f'(η) + H'(η)
CADD=2.0
1 unit (cm) on y-AXIS = 1.94 x 10^8 OF THE SAME UNITS (cm) · C2
1 η ≈ 1.8 x 10^16 UNITS ON THE X-AXIS · C
PREPARED BY AMJ-SK
x
η

Plate No. 2


f′(η), H′(η)
f′(η) + H′(η)
f′(η)
H′(η)
Report No(IV), Eqs. (3.2.7) to (3.2.9); f′(η) + H′(η)
CADD-3.0
1 UNIT (CM) ON Y-AXIS = 1.411 × 10^5 OF THE SAME UNITS (CM) · C₂
1 η = 1.8 × 10^16 UNITS ON THE X-AXIS · C₂
PREPARED BY RMJ-SK
X
η
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.8
Plate No. 3

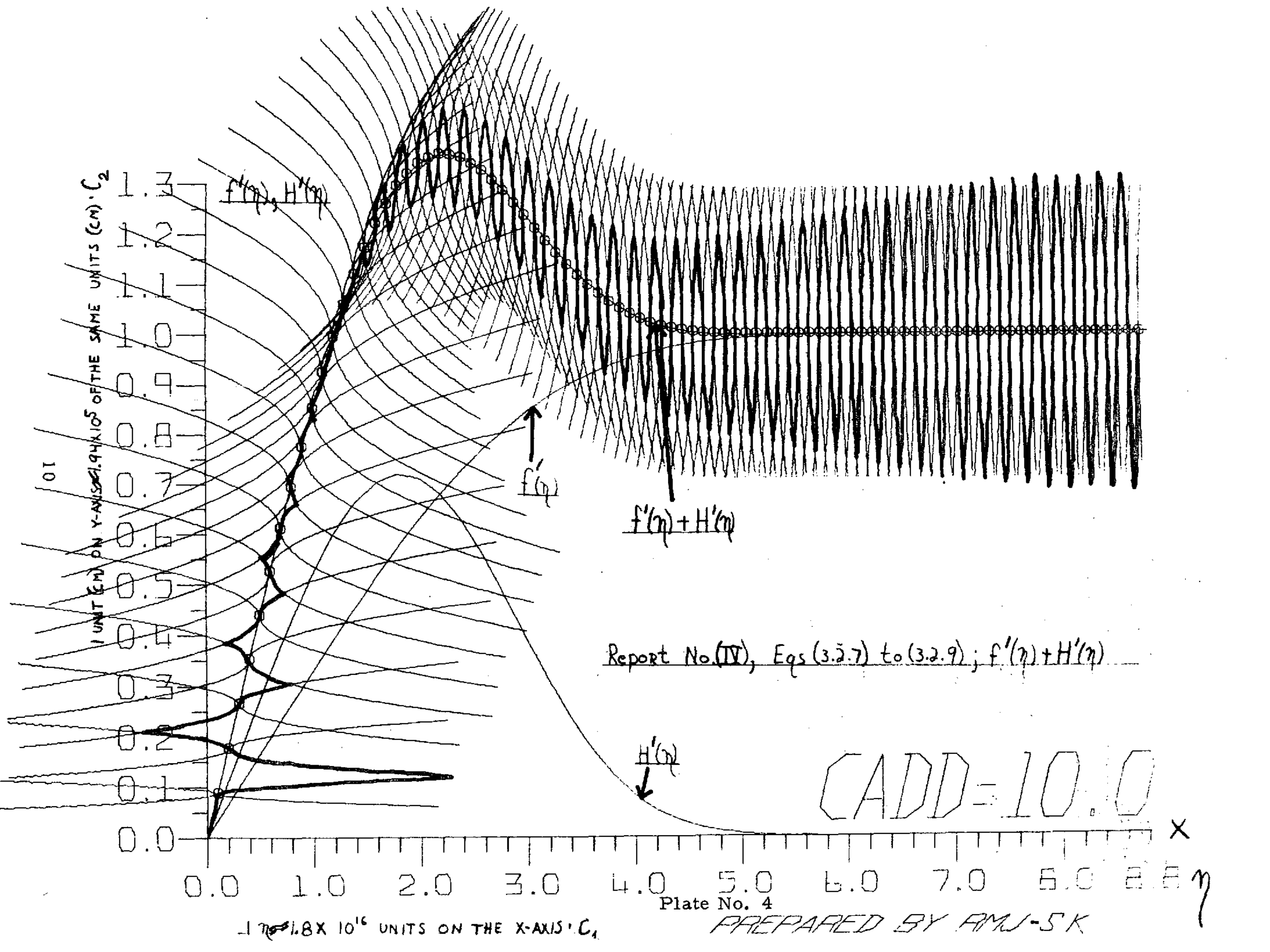
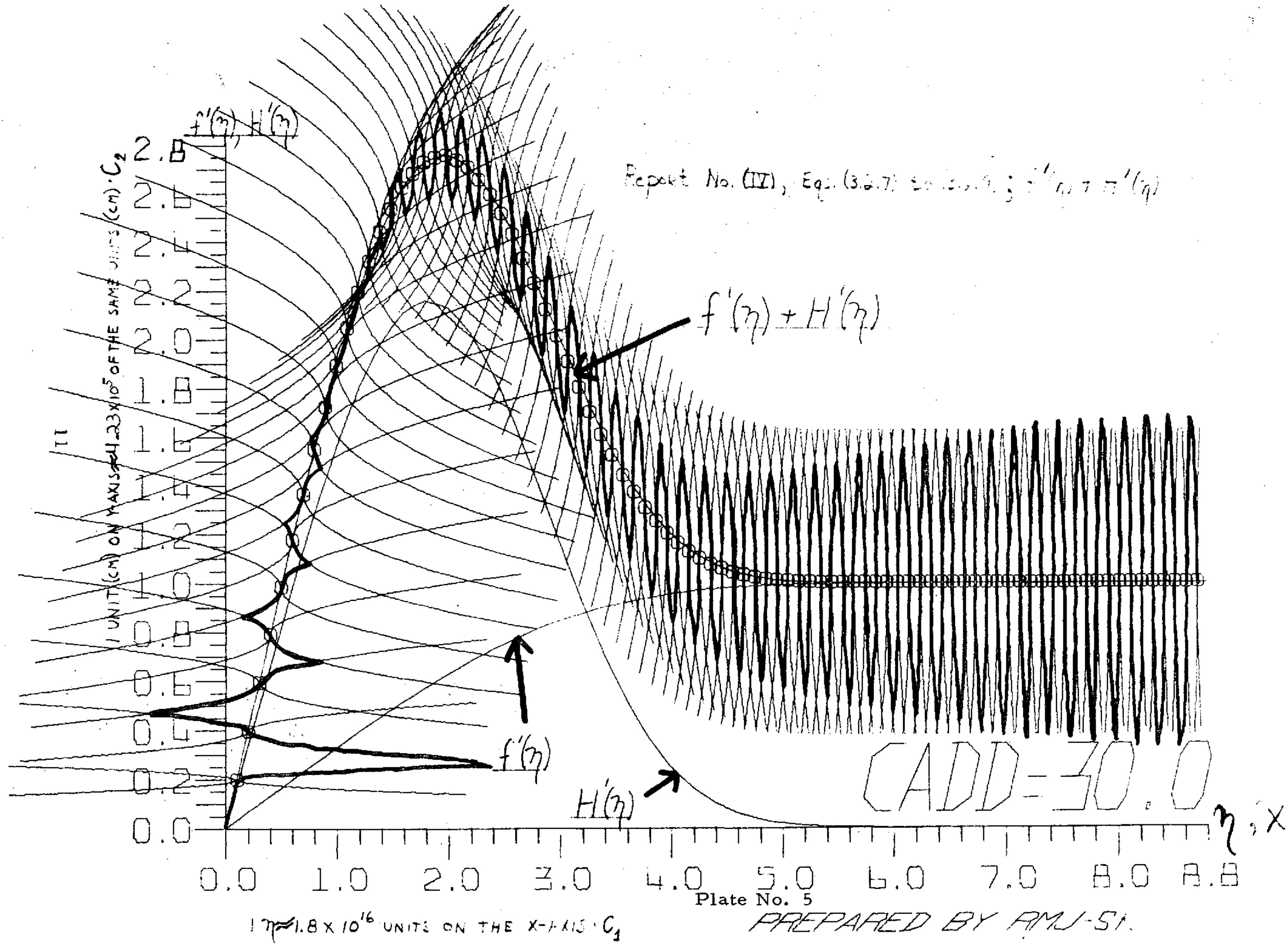

1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
f'(η), H'(η)
1 UNIT (CM) ON Y-AXIS=1.94X10^5 OF THE SAME UNITS (CM) · C2
10
f'(η)
f'(η)+H'(η)
Report No.(IV), Eqs (3.2.7) to (3.2.9); f'(η)+H'(η)
H'(η)
CADD=10.0
X
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
η
1 η=1.8X 10^16 UNITS ON THE X-AXIS · C1
PREPARED BY AMJ-SK

Plate No. 4



Report No. (IV), Eqs. (3.2.7)
f'(η), H'(η)
1 UNIT (CM) ON Y-AXIS ≈ 4.23 X 10^5 OF THE SAME UNITS (cm) · C2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
f'(η) + H'(η)
f'(η)
H'(η)
CADD=30.0
η ; X
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 8.8
1η ≈ 1.8 X 10^16 UNITS ON THE X-AXIS · C1
PREPARED BY AMJ-S1

Plate No. 5

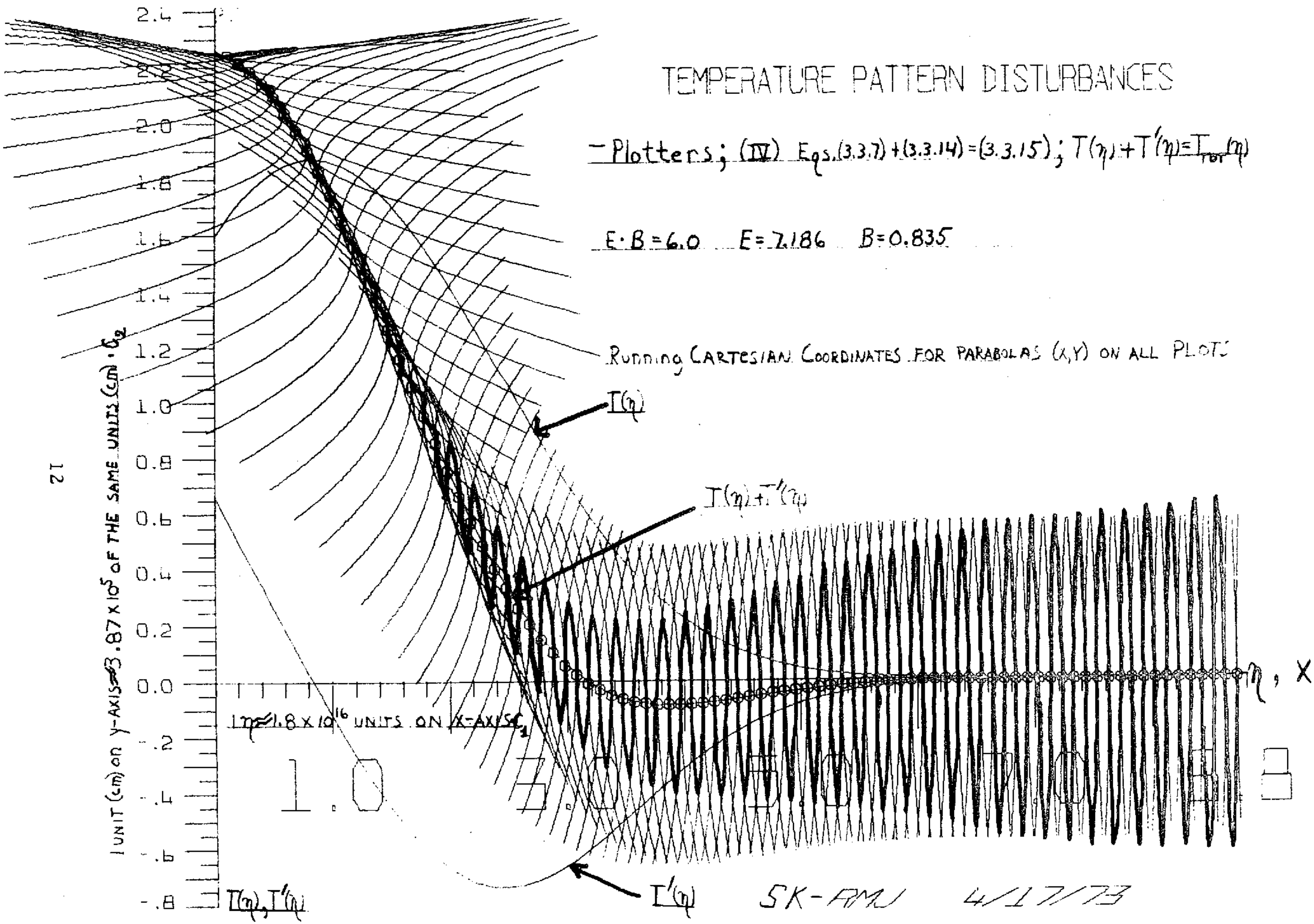
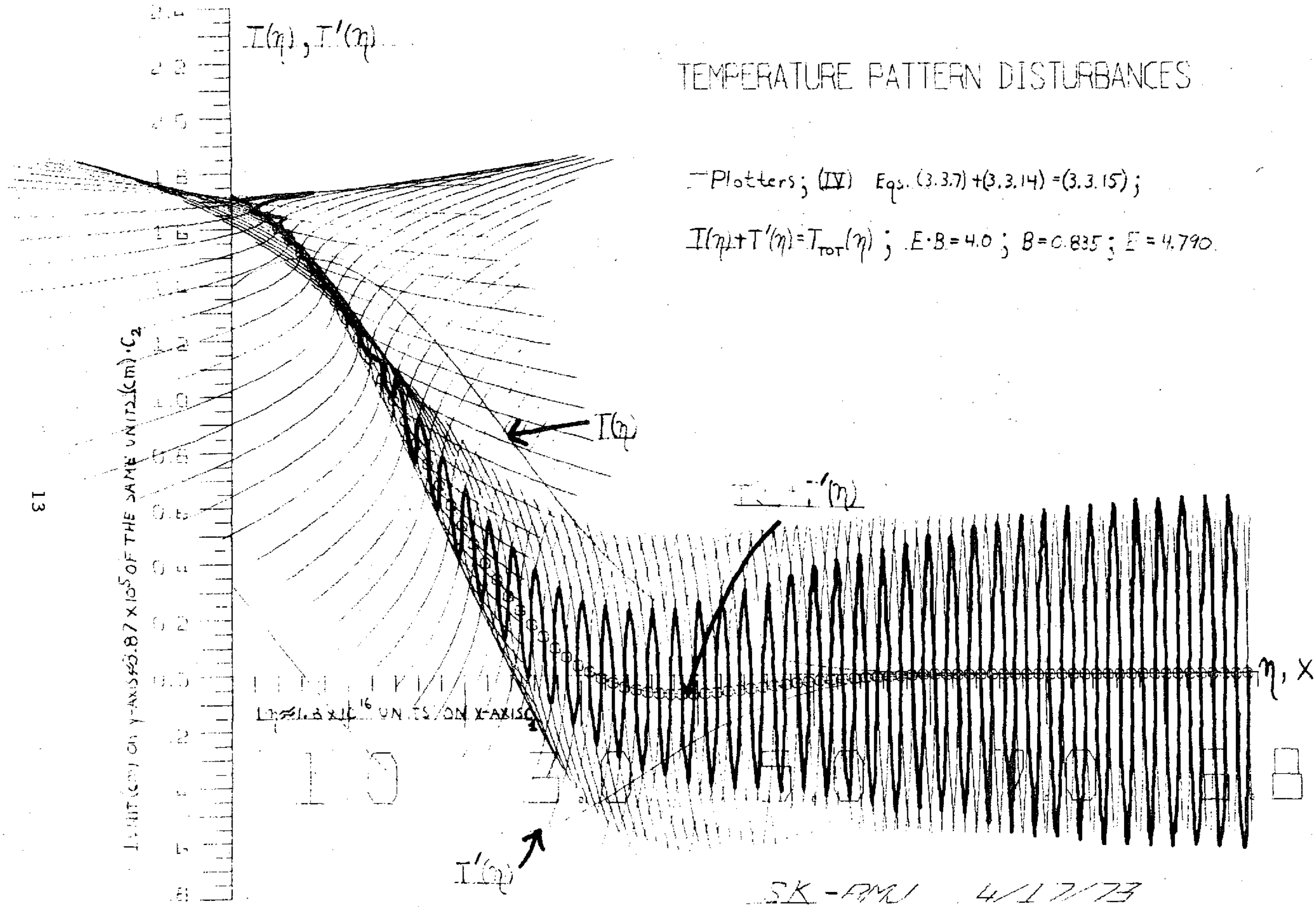

TEMPERATURE PATTERN DISTURBANCES
- Plotters; (IV) Eqs.(3.3.7)+(3.3.14)=(3.3.15); T(η)+T′(η)=T_TOT(η)
E·B=6.0 E=7.186 B=0.835
Running Cartesian Coordinates for Parabolas (x,y) on all plots
T(η)
T(η)+T′(η)
T′(η)
1 unit (cm) on y-axis = 3.87 x 10^5 of the same units (cm) · G2
1η = 1.8 x 10^16 units on x-axis · G1
η, X
T(η), T′(η)
SK-AMJ 4/17/73
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-.2
-.4
-.6
-.8

Plate No. 6



TEMPERATURE PATTERN DISTURBANCES
Plotters; (IV) Eqs. (3.3.7)+(3.3.14)=(3.3.15);
I(η)+T'(η)=T_TOT(η); E·B=4.0; B=0.835; E=4.790
I(η), T'(η)
I(η)
T'(η)
η, X
1η ≈ 1.8 x 10^16 UNITS ON X-AXIS
OF THE SAME UNITS (cm)·C2
SK-RMJ 4/17/73

Plate No. 7

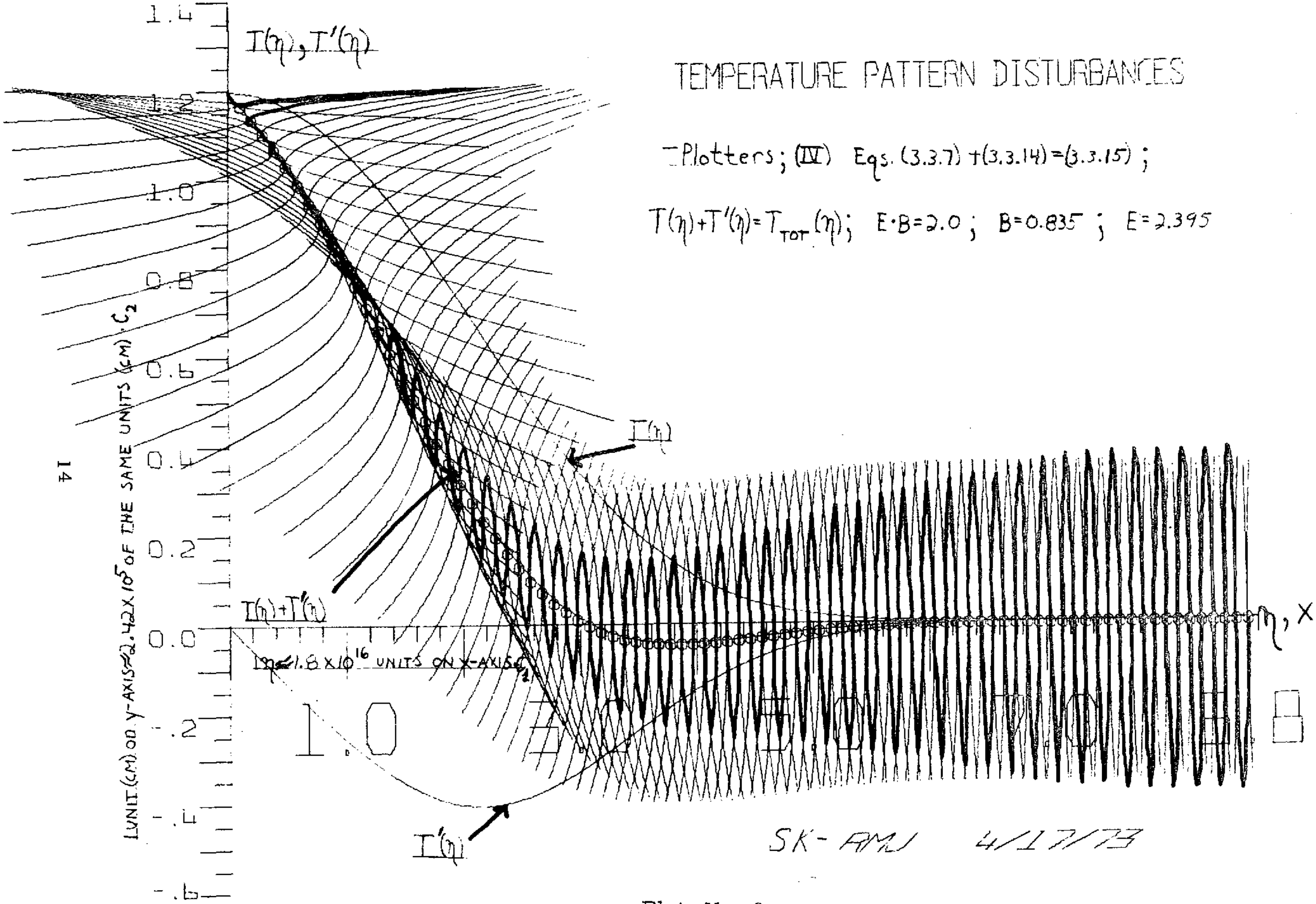

TEMPERATURE PATTERN DISTURBANCES
Plotters; (IV) Eqs. (3.3.7) + (3.3.14) = (3.3.15);
T(η) + T'(η) = T_TOT(η); E·B = 2.0; B = 0.835; E = 2.395
T(η), T'(η)
T(η)
T(η) + T'(η)
T'(η)
1η ≈ 1.8 x 10^16 UNITS ON X-AXIS C1
1 UNIT (CM) ON Y-AXIS ≈ 2.42 x 10^5 OF THE SAME UNITS (CM) C2
η, X
SK-AMJ 4/17/73

Plate No. 8

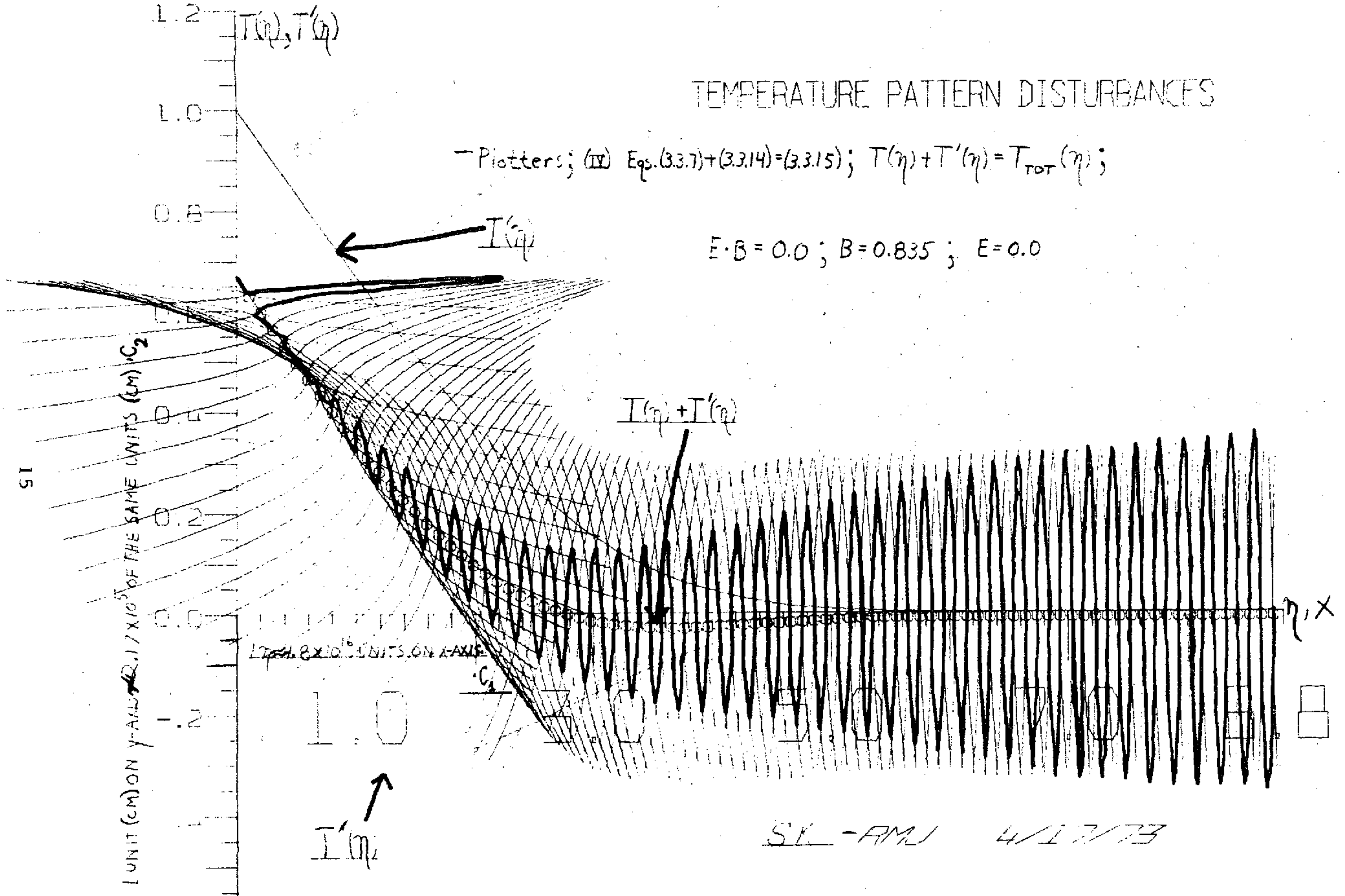

TEMPERATURE PATTERN DISTURBANCES
-Plotters; (IV) Eqs.(3.3.7)+(3.3.14)=(3.3.15); T(η)+T′(η)=T_TOT(η);
E·B=0.0; B=0.835; E=0.0
T(η), T′(η)
T(η)
T(η)+T′(η)
T′(η)
η, X
1.2
1.0
0.8
0.4
0.2
0.0
-.2
1 UNIT (CM) ON Y-AXIS 2.1/X10^5 OF THE SAME UNITS (CM)·C2
1 INCH 4.8X10^16 UNITS ON X-AXIS ·C1
1.0
SK-AMJ 4/17/73

Plate No. 9

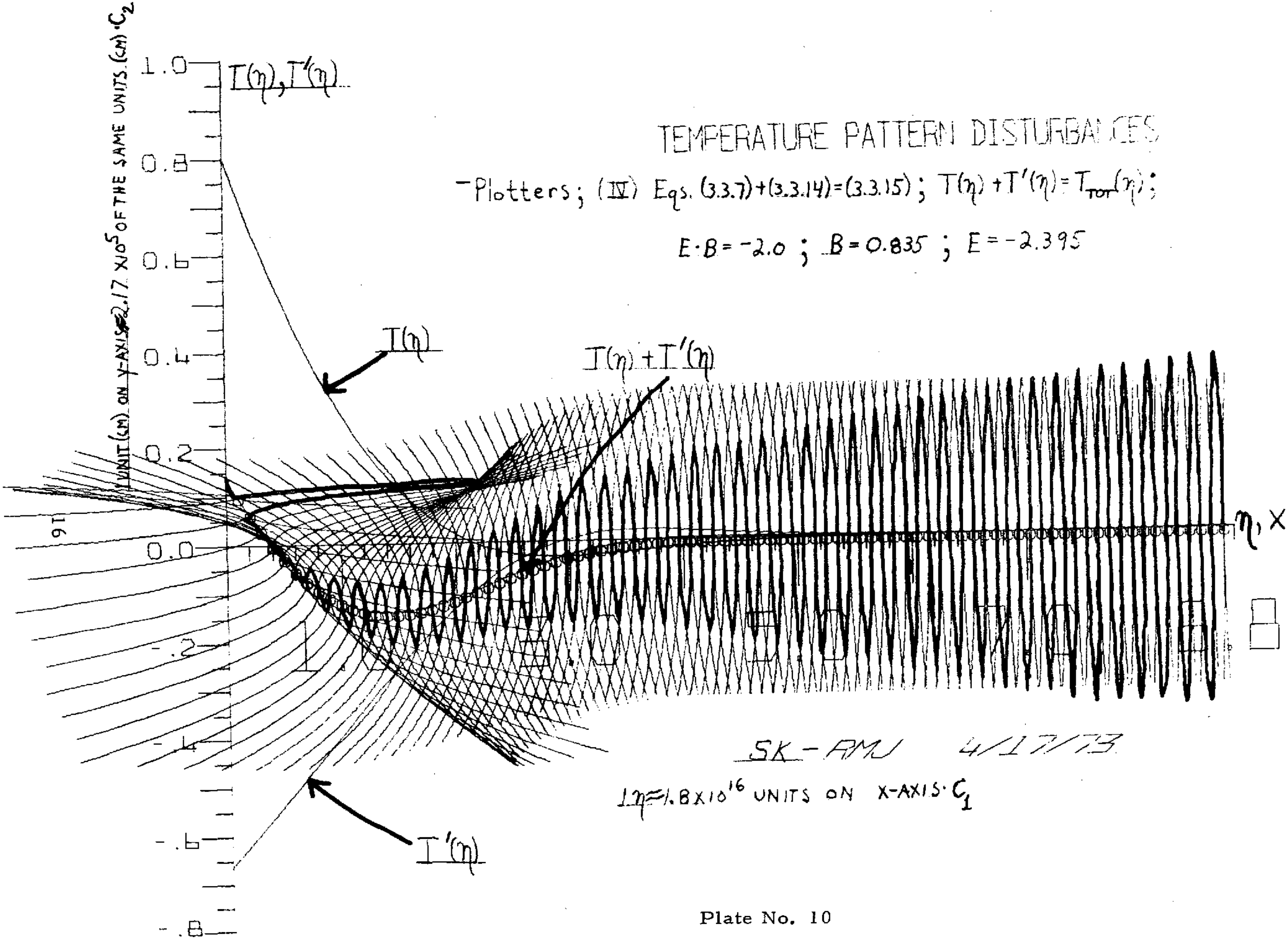

TEMPERATURE PATTERN DISTURBANCES
-Plotters; (IV) Eqs. (3.3.7)+(3.3.14)=(3.3.15); T(η) + T′(η) = T_TOT(η);
E·B = -2.0 ; B = 0.835 ; E = -2.395
T(η), T′(η)
UNIT (CM) ON Y-AXIS = 2.17 X10^5 OF THE SAME UNITS (CM)·C_2
1.0
0.8
0.6
0.4
0.2
0.0
-.2
-.4
-.6
-.8
T(η)
T(η) + T′(η)
T′(η)
η, X
SK-AMJ 4/17/73
1η = 1.8X10^16 UNITS ON X-AXIS·C_1

Plate No. 10

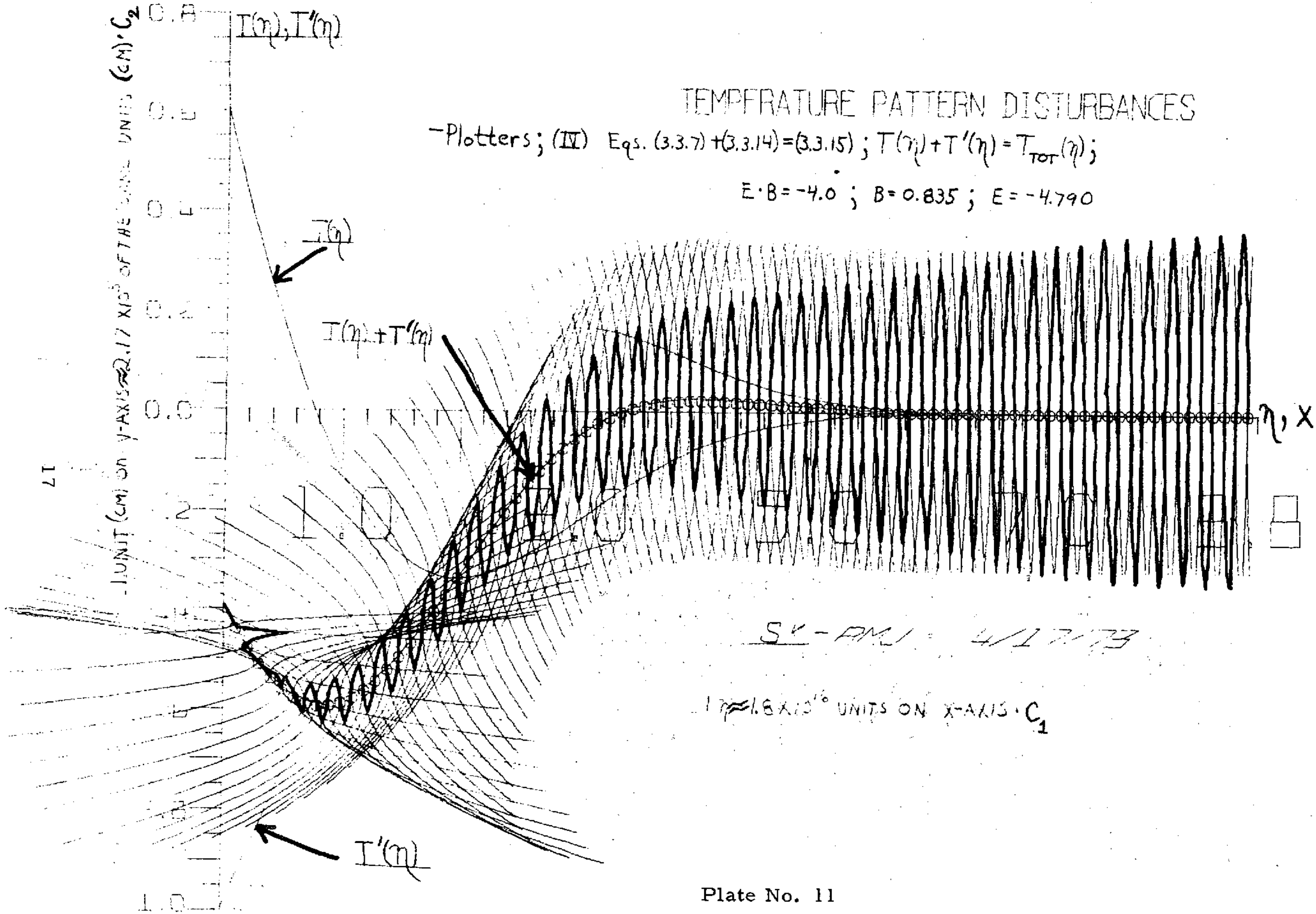


Plate No. 11

17

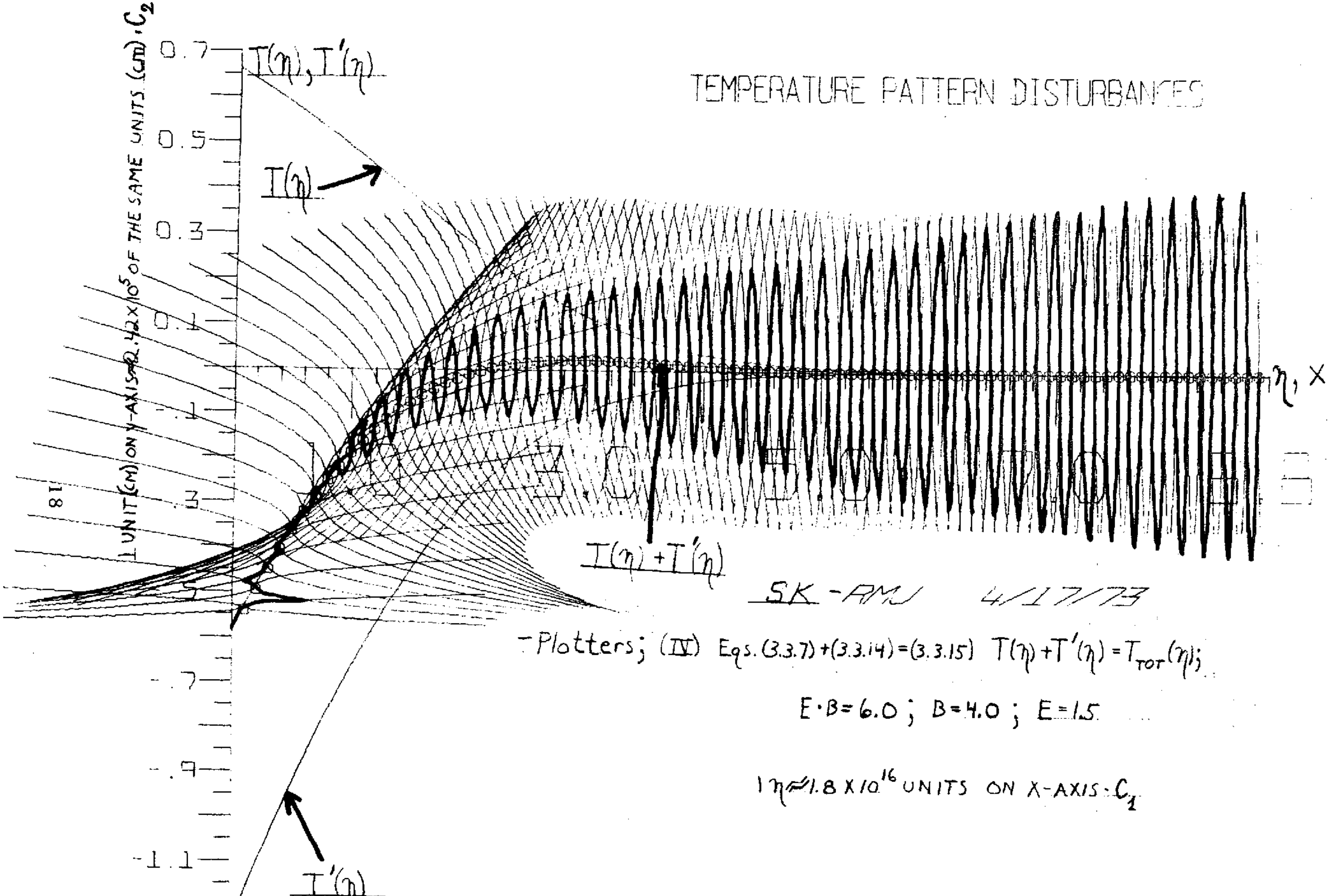

TEMPERATURE PATTERN DISTURBANCES
1 UNIT (CM) ON Y-AXIS ≈ 2.42 X 10^5 OF THE SAME UNITS (cm) · C_2
T(η), T′(η)
T(η)
T(η) + T′(η)
T′(η)
η, X
0.7
0.5
0.3
0.1
-.1
-.3
-.5
-.7
-.9
-1.1
-1.3
SK-AMJ 4/17/73
-Plotters; (IV) Eqs. (3.3.7)+(3.3.14)=(3.3.15) T(η)+T′(η) = T_TOT(η);
E·B=6.0 ; B=4.0 ; E=1.5
1η ≈ 1.8 X 10^16 UNITS ON X-AXIS · C_1

Plate No. 12

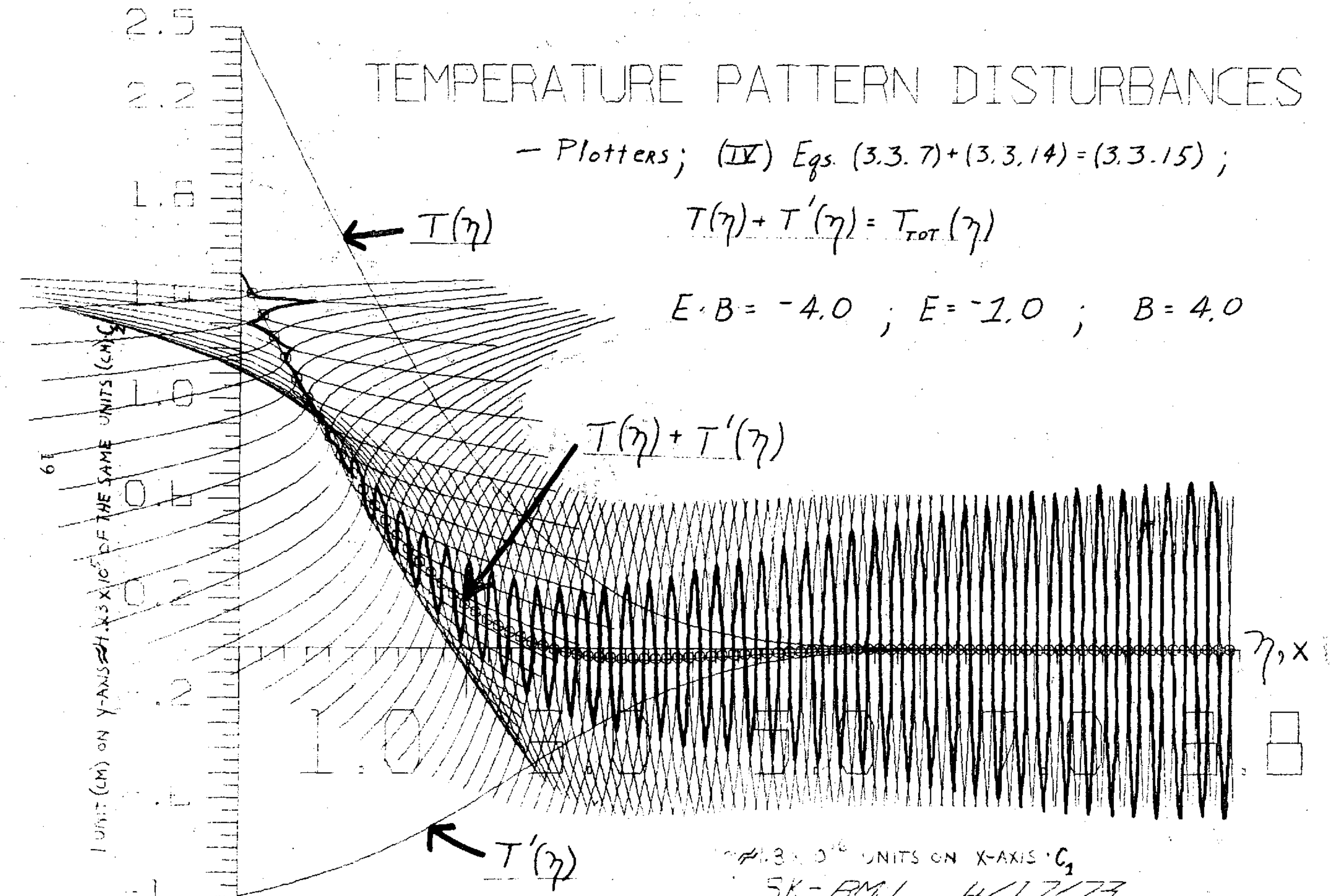
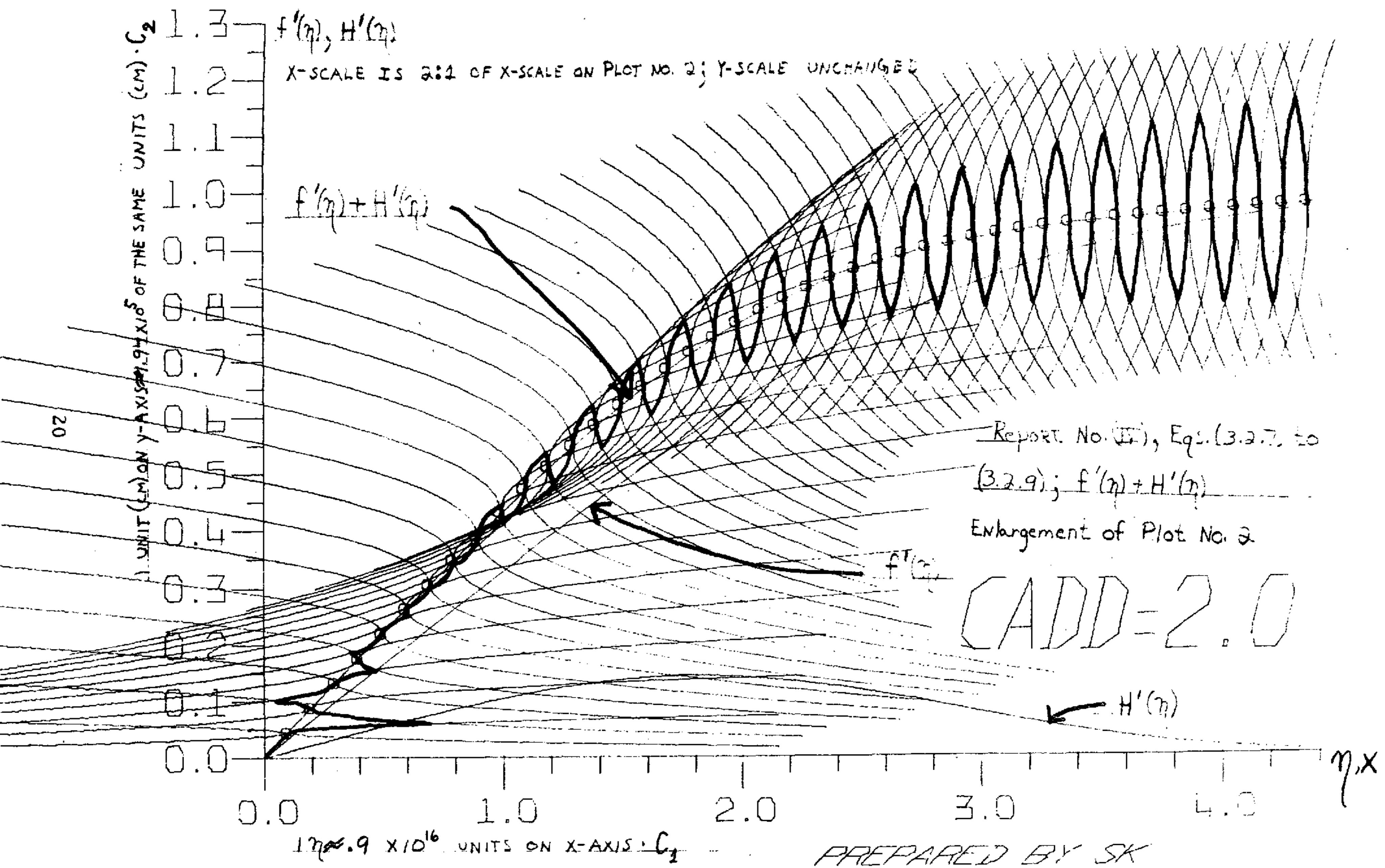

TEMPERATURE PATTERN DISTURBANCES
– Plotters; (IV) Eqs. (3.3.7) + (3.3.14) = (3.3.15);
T(η) + T'(η) = T_TOT(η)
E·B = −4.0 ; E = −1.0 ; B = 4.0
T(η)
T(η) + T'(η)
T'(η)
η, X
2.5
2.2
1.8
1.0
0.6
0.2
.2
.6
-1.
UNITS ON X-AXIS
SK-RMJ 4/17/73

Plate No. 13



f′(η), H′(η)
X-SCALE IS 2:1 OF X-SCALE ON PLOT NO. 2; Y-SCALE UNCHANGED
1 UNIT (CM) ON Y-AXIS ≈1.94 X10^5 OF THE SAME UNITS (CM) · C₂
f′(η) + H′(η)
Report No. (II), Eqs. (3.2.7) to (3.2.9); f′(η) + H′(η)
Enlargement of Plot No. 2
f′(η)
CADD=2.0
H′(η)
η, X
1η ≈ .9 X10^16 UNITS ON X-AXIS · C₁
PREPARED BY SK
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.0
2.0
3.0
4.0

Plate No. 14

VELOCITY DISTRIBUTION IN THE BOUNDRY LAYER ALONG A PLATE

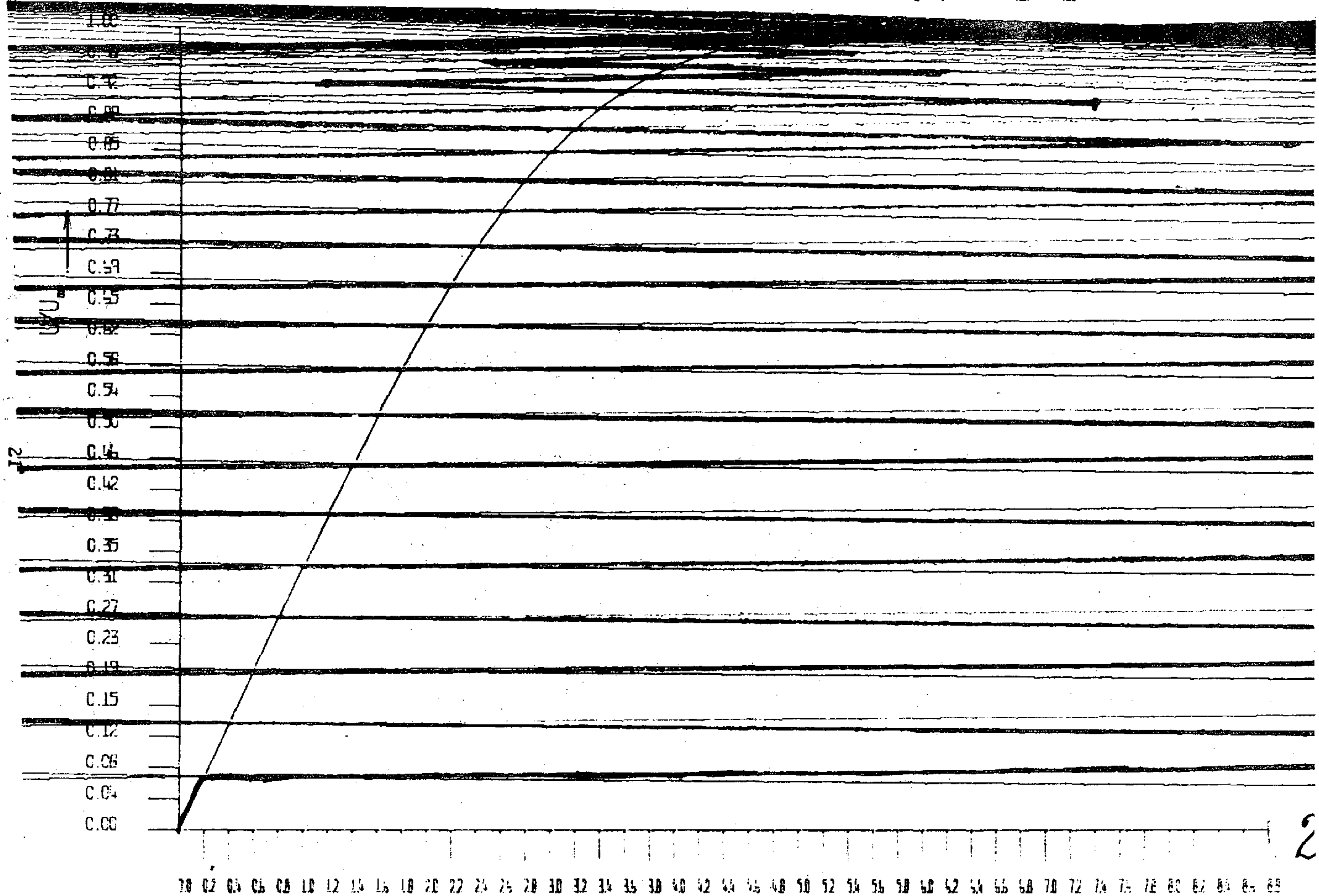


Plate No. 21

PREPARED BY D WIERENGA

ETA →

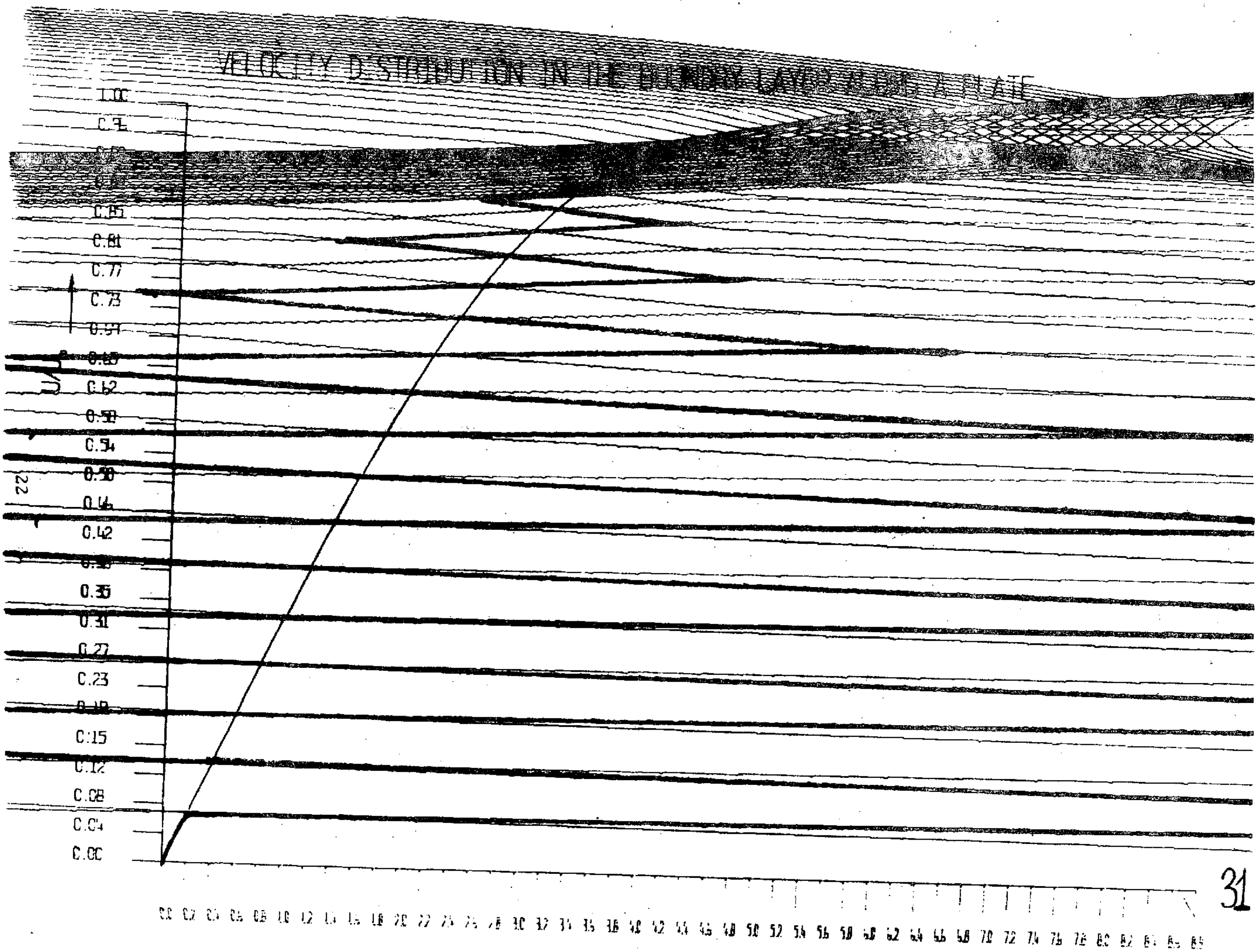

VELOCITY DISTRIBUTION IN THE BOUNDARY LAYER ALONG A PLATE
31

Plate No. 31

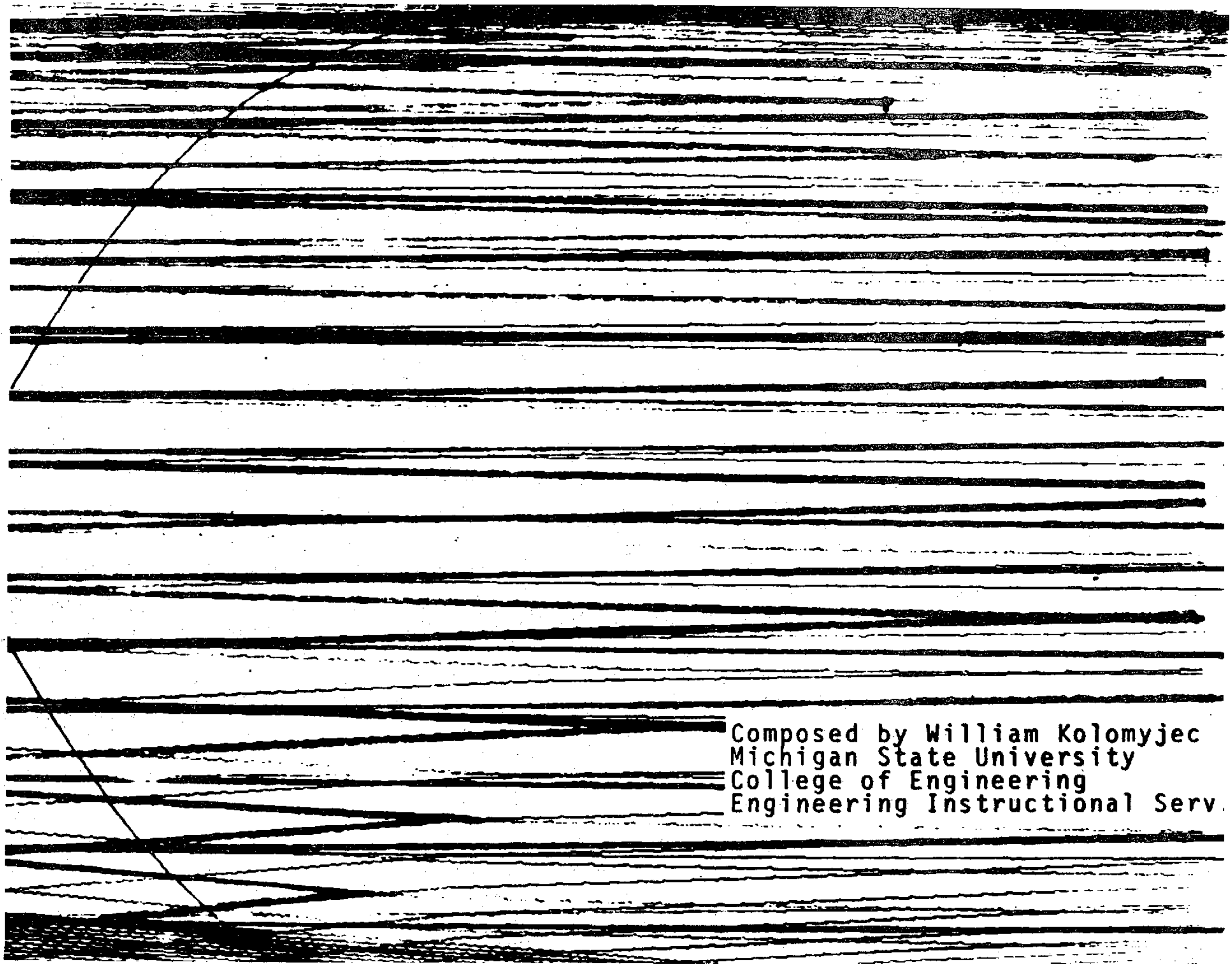
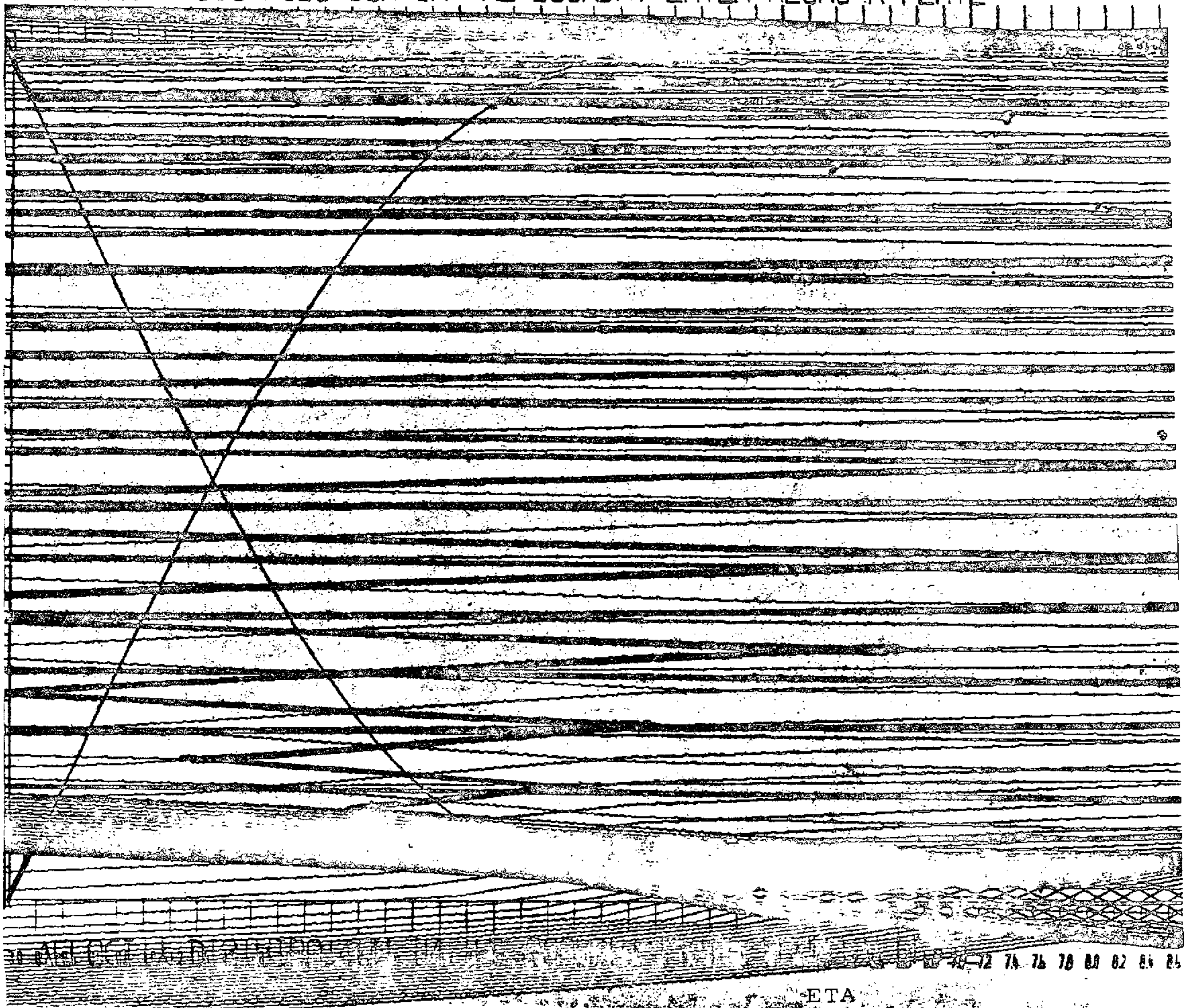

Composed by William Kolomyjec
Michigan State University
College of Engineering
Engineering Instructional Serv.

Plate No. 30

Plate No. 35

24

VELOCITY DISTRIBUTION IN THE BOUNDARY LAYER ALONG A PLATE



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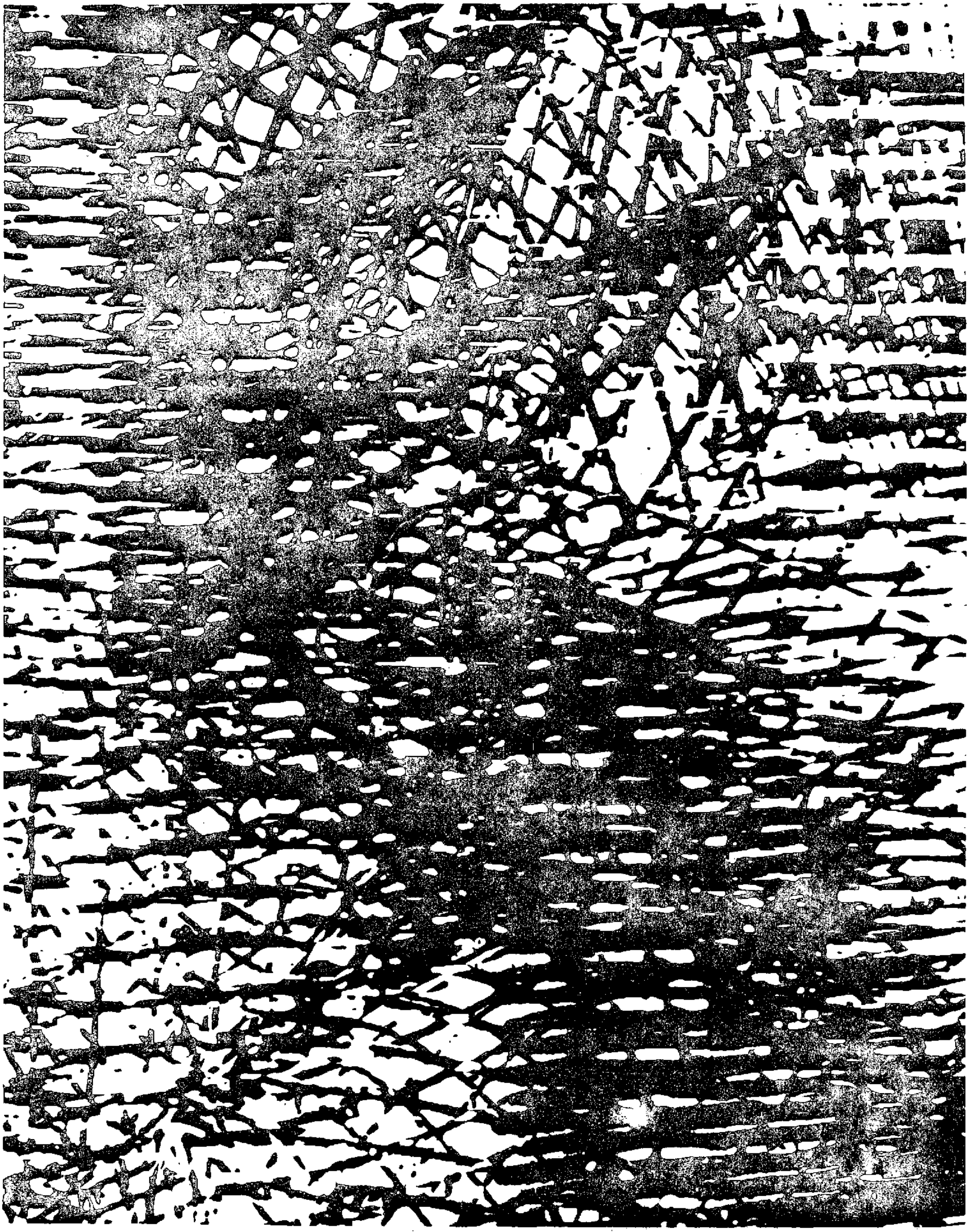

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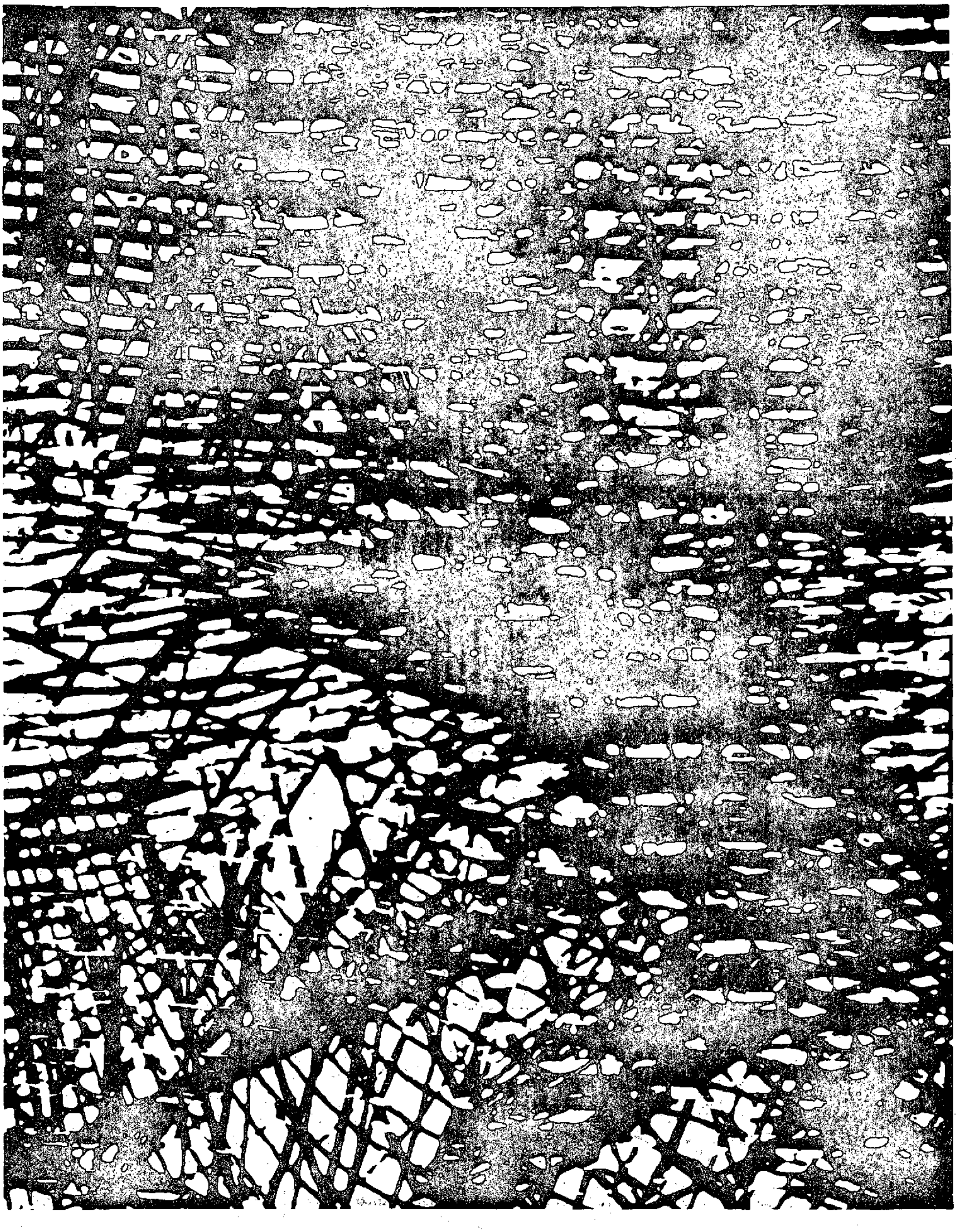

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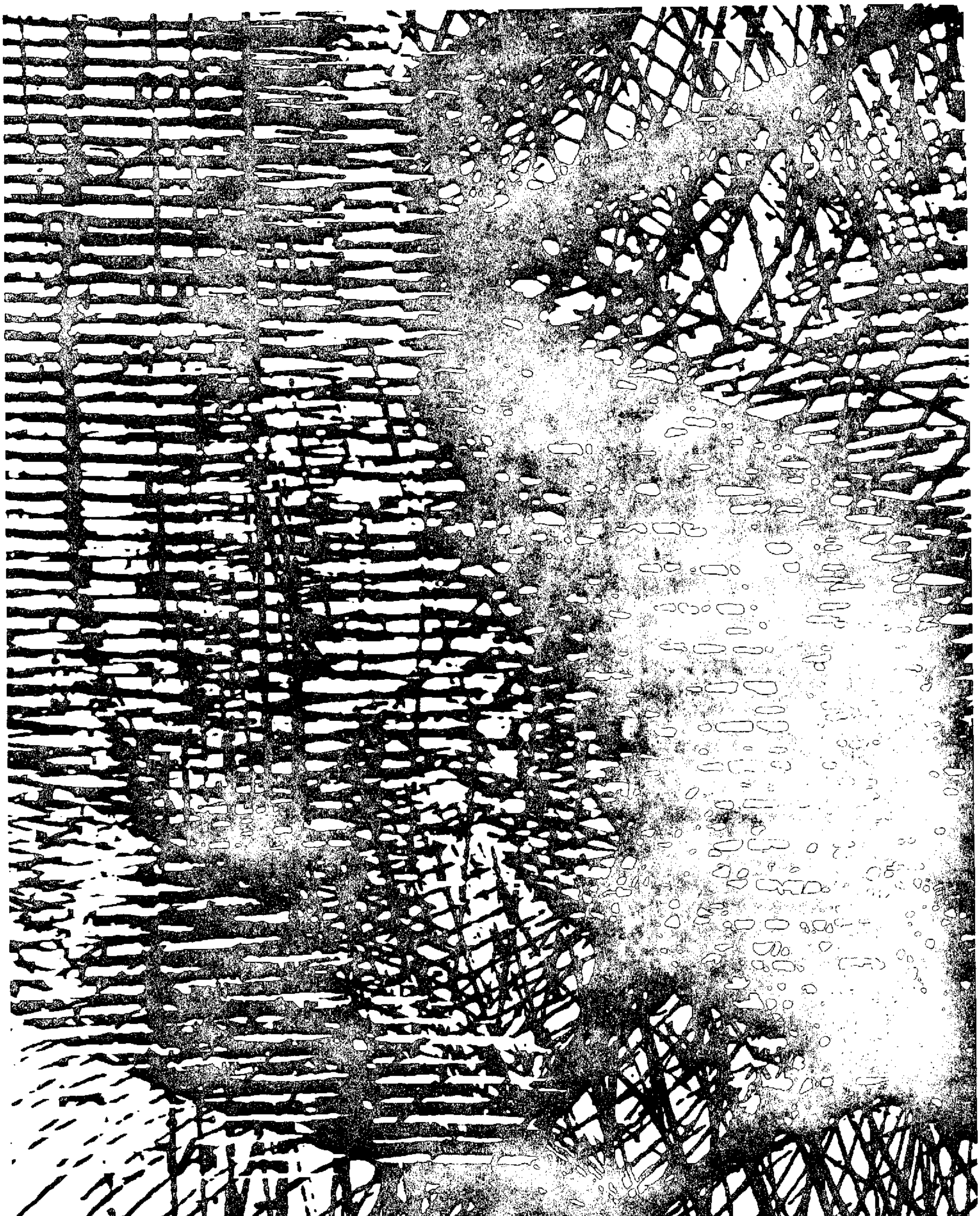

Plate No. 43

Plate No. 44